

International Journal of Modern Physics E  
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## The internal structure of jets at colliders: light and heavy quark inclusive hadronic distributions

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Received (received date)

Revised (revised date)

In this paper, we report our results on charged hadron multiplicities of heavy quark initiated jets produced in high energy collisions. After implementing the so-called dead cone effect in QCD evolution equations, we find that the average multiplicity decreases significantly as compared to the massless case. Finally, we discuss on the transverse momentum distribution of light quark initiated jets and emphasize on the comparison between our predictions and CDF data.

### 1. Introduction

High- $p_t$  jets can be initiated either in a short-distance interaction among partons in high energy collisions such as  $pp$ ,  $p\bar{p}$ , in the DIS  $e^\pm p$ , the  $e^+e^-$  annihilation and via electroweak (or new physics) processes.

In this framework, we compute the average (charged) multiplicity of a jet initiated by a heavy quark. For this purpose, we extend the modified leading logarithmic approximation (MLLA) evolution equations<sup>1</sup> to the case where the jet is initiated by a heavy (charm, bottom) quark. The average multiplicity of light quark jets produced in high energy collisions can be written as  $N(Y) \propto \exp \left\{ \int^Y \gamma(y) dy \right\}$  ( $Y$  and  $y$  are defined in section 2), where  $\gamma \simeq \gamma_0 + \Delta\gamma$  is the anomalous dimension that accounts for soft and collinear gluons in the double logarithmic approximation (DLA)  $\gamma_0 \simeq \sqrt{\alpha_s}$ , in addition to hard collinear gluons  $\Delta\gamma \simeq \alpha_s$  or single logarithms (SLs), which better account for energy conservation and the running of the coupling constant  $\alpha_s$ <sup>1</sup>.

The inclusive  $k_\perp$ -distribution of particles inside a jet has been computed at MLLA accuracy in the limiting spectrum approximation<sup>2</sup>, *i.e.* assuming an infrared cutoff  $Q_0$  equal to  $\Lambda_{QCD}$  (for a review, see also<sup>1</sup>). MLLA corrections, of relative magnitude  $\mathcal{O}(\sqrt{\alpha_s})$  with respect to the leading double logarithmic approximation (DLA), were shown to be quite substantial<sup>2</sup>. Therefore, we also included corrections of order  $\mathcal{O}(\alpha_s)$ , that is next-to-next-to-leading or Next-to-MLLA (NMLLA).

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Lastly, both observables, the average multiplicity and the inclusive  $k_\perp$ -distribution are computed under the assumption of local parton hadron duality (LPHD) as hadronization model<sup>3,4</sup>.

## 2. Kinematics and variables

As known from jet calculus for light quarks, the evolution time parameter determining the structure of the parton branching of the primary gluon is given by (for a review see<sup>1</sup> and references therein)

$$y = \ln \left( \frac{k_\perp}{Q_0} \right), \quad k_\perp = zQ \geq Q_0, \quad Q = E\Theta \geq Q_0, \quad (1)$$

where  $k_\perp$  is the transverse momentum of the gluon emitted off the light quark,  $Q$  is the virtuality of the jet (or jet hardness),  $E$  the energy of the leading parton,  $Q_0/E \leq \Theta \leq \Theta_0$  is the emission angle of the gluon ( $\Theta \ll 1$ ),  $\Theta_0$  the total half opening angle of the jet being fixed by experimental requirements. Let us define in this context the variable  $Y$  as  $y = Y + \ln z$ ,  $Y = \ln \left( \frac{Q}{Q_0} \right)$ . The appearance of this scale is a consequence of angular ordering (AO) of successive parton branchings in QCD cascades<sup>1,3</sup>.

The corresponding evolution time parameter for a jet initiated by a heavy quark with energy  $E$  and mass  $m$  appears in a natural way and reads  $\tilde{y} = \ln \left( \frac{\kappa_\perp}{Q_0} \right)$ ,  $\kappa_\perp^2 = k_\perp^2 + z^2 m^2$ <sup>5</sup>, which for collinear emissions  $\Theta \ll 1$  can also be rewritten in the form

$$\kappa_\perp = z\tilde{Q}, \quad \tilde{Q} = E(\Theta^2 + \Theta_m^2)^{\frac{1}{2}}, \quad (2)$$

with  $\Theta \geq \Theta_m$  (see Fig. 1). An additional comment is in order concerning the AO for gluons emitted off the heavy quark. In (2),  $\Theta$  is the emission angle of the primary gluon  $g$  being emitted off the heavy quark. Now let  $\Theta'$  be the emission angle of a second gluon  $g'$  relative to the primary gluon with energy  $\omega' \ll \omega$  and  $\Theta''$  the emission angle relative to the heavy quark; in this case the *incoherence* condition  $\Theta'^2 \leq (\Theta^2 + \Theta_m^2)$  together with  $\Theta'' > \Theta_m$  (the emission angle of the second gluon should still be larger than the dead cone) naturally leads (2) to become the proper evolution parameter for the gluon subjet (for more details see<sup>5</sup>). For  $\Theta_m = 0$ , the standard AO ( $\Theta' \leq \Theta$ ) is recovered. Therefore, for a massless quark, the virtuality of the jet simply reduces to  $Q = E\Theta$  as given above. The same quantity  $\kappa_\perp$  determines the scale of the running coupling  $\alpha_s$  in the gluon emission off the heavy quark. It can be related to the anomalous dimension of the process by

$$\gamma_0^2(\kappa_\perp) = 2N_c \frac{\alpha_s(\kappa_\perp)}{\pi} = \frac{1}{\beta_0(\tilde{y} + \lambda)}, \quad \beta_0(n_f) = \frac{1}{4N_c} \left( \frac{11}{3}N_c - \frac{2}{3}n_f \right), \quad \lambda = \ln \frac{Q_0}{\Lambda_{QCD}}, \quad (3)$$

where  $n_f$  is the number of active flavours and  $N_c$  the number of colours. The variation of the effective coupling  $\alpha_s$  as  $n_f \rightarrow n_f + 1$  over the heavy quarks threshold has been suggested by next-to-leading (NLO) calculations in the  $\overline{MS}$  scheme<sup>4</sup> and

is sub-leading in this frame. In this context  $\beta_0(n_f)$  will be evaluated at the total number of quarks we consider in our application. The four scales of the process are related as follows,

$$\tilde{Q} \gg m \gg Q_0 \sim \Lambda_{QCD},$$

where  $Q_0 \sim \Lambda_{QCD}$  corresponds to the limiting spectrum approximation<sup>1</sup>. Finally, the dead cone phenomenon imposes the following bounds of integration to the perturbative regime

$$\frac{m}{\tilde{Q}} \leq z \leq 1 - \frac{m}{\tilde{Q}}, \quad m^2 \leq \tilde{Q}^2 \leq E^2(\Theta_0^2 + \Theta_m^2), \quad (4)$$

which now account for the phase-space of the heavy quark jet. The last inequality states that the minimal transverse momentum of the jet  $\tilde{Q} = E\Theta_m = m$  is given by the mass of the heavy quark, which enters the game as the natural cut-off parameter of the perturbative approach.

### 3. QCD evolution equations

The system of QCD evolution equations for the heavy quark initiated jet, corresponding to the process described in Fig. 1 is found to read<sup>6</sup>

$$\frac{N_c}{C_F} \frac{dN_Q}{d\tilde{Y}} = \epsilon_1(\tilde{Y}, L_m) \int_{\tilde{Y}_m}^{\tilde{Y}_{ev}} d\tilde{y} \gamma_0^2(\tilde{y}) N_g(\tilde{y}) - \epsilon_2(\tilde{Y}, L_m) \gamma_0^2(\tilde{Y}) N_g(\tilde{Y}), \quad (5)$$

$$\frac{dN_g}{d\tilde{Y}} = \int_{\tilde{Y}_m}^{\tilde{Y}_{ev}} d\tilde{y} \gamma_0^2(\tilde{y}) N_g(\tilde{y}) - A(\tilde{Y}, L_m) \gamma_0^2(\tilde{Y}) N_g(\tilde{Y}), \quad (6)$$

where  $A(\tilde{Y}, L_m)$ ,  $a(n_f)$ ,  $\epsilon_1(\tilde{Y}, L_m)$  and  $\epsilon_2(\tilde{Y}, L_m)$  are defined in<sup>6</sup>. As a consistency

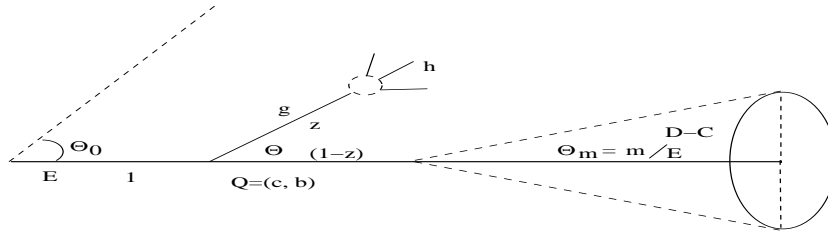


Fig. 1. Parton splitting in the process  $Q \rightarrow Qg$ : a dead cone with opening angle  $\Theta_m$  is schematically shown.

check, upon integration over  $\tilde{Y}$  of the DLA term in Eq.(5), the phase space structure of the radiated quanta reads

$$N_Q(\ln \tilde{Q}) \approx 1 + \frac{C_F}{N_c} \int_0^{\Theta_0^2} \frac{\Theta^2 d\Theta^2}{(\Theta^2 + \Theta_m^2)^2} \int_{m/\tilde{Q}}^{1-m/\tilde{Q}} \frac{dz}{z} [\gamma_0^2 N_g] (\ln z \tilde{Q}). \quad (7)$$

Notice that the lower bound over  $\Theta^2$  in (7) ( $\tilde{Y}$  in (5)) can be taken down to “0” ( $Y_m = L_m$  in (5)) because the heavy quark mass plays the role of collinear cut-off parameter.

#### 4. Phenomenological consequences

Working out the structure of (5) and (6), we obtain the rough difference between the light and heavy quark jet multiplicities, which yields,

$$N_q - N_Q \stackrel{E \rightarrow \infty}{\approx} \left[ 1 - \exp \left( -2 \sqrt{\frac{L_m}{\beta_0}} \right) \right] N_q, \quad N_q \propto \exp 2 \sqrt{\frac{\tilde{Y}}{\beta_0}}. \quad (8)$$

It can be seen that (8) is exponentially increasing because it is dominated by the leading DLA energy dependence of  $N_q$ . According to (8), the gap arising from the dead cone effect should be bigger for the  $b$  than for the  $c$  quark at the primary state bremsstrahlung radiation off the heavy quark jet. The approximated solution of the evolution equations leads to the rough behaviour of  $N_q - N_Q$  in (8), which is not exact in its present form. That is why, in the following, we use the numerical solution of the equations (5) and (6). In this study we advocate the role of mean multiplicities

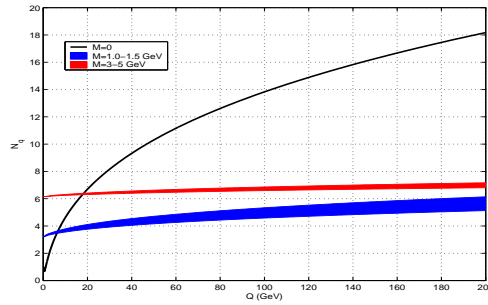


Fig. 2. Massless and massive quark jet average multiplicity  $N_Q^{total}$  including heavy quark flavour decays as a function of the jet hardness  $Q$ . Bands indicate  $m_c$  and  $m_b$  in the  $[1, 1.5]$  and  $[3, 5]$  GeV intervals respectively.

of jets as a potentially useful signature for  $b$ -tagging and new associated physics<sup>7</sup> when combined with other selection criteria. In Fig. 2, we plot as function of the jet hardness, the total average jet multiplicity ( $N_A^{total}$ ), which accounts for the primary state radiation off the heavy quark ( $N_A^{ch}$ ) together with the decay products from the final-state flavoured hadrons ( $N_A^{dc}$ ), in the form  $N_A^{total} = N_A^{ch} + N_A^{dc}$ . Moreover, the flavour decays constants  $N_c^{dc} = 2.60 \pm 0.15$  and  $N_b^{dc} = 5.55 \pm 0.09$ <sup>8</sup> are independent of the hard process inside the cascade, such that  $N_A^{dc}$  can be added in the whole energy range. After accounting for the weak decay multiplicities, it turns out that the  $b$  quark jet multiplicity becomes slightly higher than the  $c$  quark jet multiplicity, although both remain suppressed because of the dead cone effect.

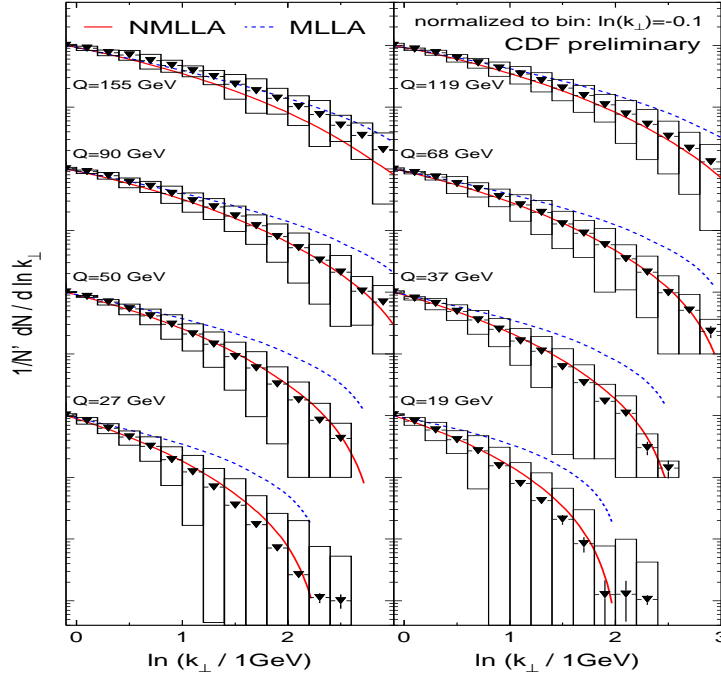


Fig. 3. CDF preliminary results for the inclusive  $k_{\perp}$  distribution at various hardness  $Q$  in comparison to MLLA and NMLLA predictions at the limiting spectrum ( $Q_0 = \Lambda_{QCD}$ ); the boxes are the systematic errors.

### 5. Single inclusive $k_{\perp}$ —distribution of charged hadrons in NMLLA

Computing the single inclusive  $k_{\perp}$ —distribution requires the definition of the jet axis. The starting point of our approach consists in considering the correlation between two particles (h1) and (h2) of energies  $E_1$  and  $E_2$  which form a relative angle  $\Theta$  inside one jet of total opening angle  $\Theta_0 > \Theta$ <sup>2</sup>. Weighting over the energy  $E_2$  of particle (h2), this relation leads to the correlation between the particle (h=h1) and the energy flux, which we identify with the jet axis<sup>2</sup>. Thus, the correlation and the relative transverse momentum  $k_{\perp}$  between (h1) and (h2) are replaced by the correlation, and transverse momentum of (h1) with respect to the direction of the energy flux. In Fig. 3<sup>9</sup>, we report measurements over a wide range of jet hardness,  $Q = E\Theta_0$ , in  $p\bar{p}$  collisions at  $\sqrt{s} = 1.96$  TeV<sup>10</sup>, together with the MLLA predictions of<sup>2</sup> and the NMLLA calculations, both at the limiting spectrum ( $\lambda = 0$ ) and taking  $\Lambda_{QCD} = 250$  MeV; the experimental distributions suffering from large normalization errors, data and theory are normalized to the same bin,  $\ln(k_{\perp}/1 \text{ GeV}) = -0.1$ . The hadronic  $k_{\perp}$ -spectrum inside a high energy jet is determined including corrections of relative magnitude  $\mathcal{O}(\sqrt{\alpha_s})$  with respect to the MLLA, in the limiting spectrum approximation. The results in the limiting spectrum approximation are

found to be in impressive agreement with measurements by the CDF collaboration, unlike what occurs at MLLA, pointing out small overall non-perturbative contributions. The MLLA predictions prove reliable in a much smaller  $k_{\perp}$  interval. At fixed jet hardness (and thus  $Y_{\Theta_0}$ ), NMLLA calculations prove accordingly trust-able in a much larger  $x$  interval.

## 6. Conclusions

Thus, our present work focusing on the differences of the average charged hadron multiplicity between jets initiated by gluons, light or heavy quarks could indeed represent a helpful auxiliary criterion to tag such heavy flavours from background for jet hardness  $Q \gtrsim 40$  GeV. Notice that we are suggesting as a potential signature the *a posteriori* comparison between average jet multiplicities corresponding to different samples of events where other criteria to discriminate heavy from light quark initiated jets were first applied. Fig.2 plainly demonstrate that the separation between light quark jets and heavy quark jets is allowed above a few tens of GeV with the foreseen errors of the experimentally measured average multiplicities of jets.

The single inclusive  $k_{\perp}$ -spectra inside a jet is determined including higher-order  $\mathcal{O}(\alpha_s)$  (i.e. NMLLA) corrections from the Taylor expansion of the MLLA evolution equations and beyond the limiting spectrum,  $\lambda \neq 0$ . The agreement between NMLLA predictions and CDF preliminary data in  $p\bar{p}$  collisions at the Tevatron is impressive, indicating very small overall non-perturbative corrections and giving further support to LPHD<sup>3</sup>.

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